thickness of the frost on the initial section, at a distance l, and the average thickness of the layer; $\delta^* = \delta_l \delta_0^{-1}$; $L = l l_0^{-1}$.

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FILTRATION OF A MAGNETIC FLUID IN A DEFORMABLE POROUS MEDIUM

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The equations of motion of a magnetizing fluid are obtained in a deformable non-magnetic porous medium.

Filtration of a magnetic fluid in nondeformable porous media was examined in [1, 2]. Derivation of the equations of magnetic fluid filtration in a deformable porous matrix consisting of deformable grains that are displaceable relative to each other is of interest.

It is assumed that an inhomogeneous magnetizing fluid fills the pore space entirely, i.e., the medium is saturated; there are no phase transitions associated with absorption (desorption) of the solid ferromagnet particles on the pore surface. The equations of fluid motion in a porous medium are obtained by local volume averaging [3] of the microequations of fluid motion in the pores, the Maxwell equation for the magnetic field in the pores and the matrix, as well as the equations of porous matrix deformation, with thermal expansion of the grains, from which the matrix consists, and the relative grain displacement taken into account. The magnetic properties of the medium as a whole (matrix + fluid) are characterized by the effective magnetic permittivity of the medium. The interphasal heat transfer between the liquid and solid phases is taken into account in the averaged heat conduction equations for the fluid and porous matrix.

The following relationships [3]

$$\langle \nabla_i f_\alpha \rangle = \nabla_i \langle f_\alpha \rangle + \sigma_{12} \langle n_{\alpha i} f_\alpha \rangle_{12}, \qquad \langle \partial_t f_\alpha \rangle = \partial_t \langle f_\alpha \rangle - \sigma_{12} \langle n_{\alpha i} u^i f_\alpha \rangle_{12}. \tag{1}$$

are used to average the microequations.

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The microequations of the fluid motion in the pores, the deformation equations of the porous matrix, and also the boundary conditions on the interphasal surface are written in the form [4-6]:

$$\partial_t \rho_\alpha v_{\alpha i} = -\nabla_k \rho_\alpha v_{\alpha i} v_\alpha^k + \nabla_k p_{\alpha i}^k + \rho_\alpha g_i, \tag{2}$$

$$\partial_t \rho_\alpha + \nabla_k \rho_\alpha v_\alpha^k = 0,\ \nabla_k B_\alpha^k = 0,\ \varepsilon^{ijk} \nabla_j H_{\alpha k} = 0,$$

$$B_{\alpha k} = H_{\alpha k} + 4\pi M_{\alpha k},$$

$$p_{1ik} = -p_1 g_{ik} + \frac{1}{4\pi} H_{1i} B_{1k} + \tau_{1ik},$$

$$p_1 = p_{01} + \frac{H_1^2}{8\pi} + \int_0^{H_1} \left[M - \rho \left(\frac{\partial M}{\partial \rho} \right)_{T,H} \right] dH,$$

$$\tau_{1ik} = 2\eta_1 e_{1ik},\ e_{1ik} = (1/2)(\nabla_i v_{1k} + \nabla_k v_{1i}),$$

$$\rho_\alpha c_\alpha \frac{dT_\alpha}{dt} = \nabla_k \varkappa_\alpha \nabla^k T_\alpha,$$

$$p_{2ik} = \lambda_2 g_{ik} \varepsilon_{2n}^n + 2\mu_2 \varepsilon_{2ik} - \beta_{2T} K_2 g_{ik} (T_2 - T_0) + \frac{1}{4\pi} \left(H_{2i} H_{2k} - \frac{1}{2} H_2^2 g_{ik} \right),$$

$$\varepsilon_{2ik} = (1/2)(\nabla_i h_{2k} + \nabla_k h_{2i}),\ v_{2i} = \frac{d_2 h_{2i}}{dt},$$

$$\{B_n\} = 0,\ \{H_{\tau i}\} = 0,\ \{v_i\} = 0,$$

$$\{p_{ik} n^k\} = 0,\ \{T\} = 0,\ \{\varkappa n^i \nabla_i T\} = 0.$$

The equations in which the quantities are denoted with the subscript 1 refer to the liquid phase, and with the subscript 2 to the solid phase, and with the subscript α (α = 1; 2) to both phases. Since the matrix is assumed nonmagnetic, we should set $M_{2k} = 0$. The difference in the specific heats for constant pressure and volume is not taken into account in (2) and henceforth. Summation is over repeated subscripts. We neglect the magnetocaloric effect as well as the work of the internal forces in the heat influx equations for the liquid and solid phases. Since the deformations within the grain (but not the relative displacements of the grain) are small, we neglect the convective term in the substantial derivative (i.e., $d_2/dt \approx \partial/\partial t$) as well as the first term in the right side of the momentum equation for the solid phase, which is the convective momentum transport. The equations of state of both phases should also be appended to (2).

Taking the average, with respect to the phases, according to (1), of the momentum equations for both phases (2) and combining them, with the boundary conditions (2) taken into account, we find the momentum equation for the mixture (matrix + fluid)

$$\partial_t (m_1 \langle \rho_1 \rangle_1 \langle v_{1i} \rangle_1 + m_2 \langle \rho_2 \rangle_2 \langle v_{2i} \rangle_2) = -\nabla_k m_1 \langle \rho_1 \rangle_1 \langle v_{1i} \rangle_1 \langle v_1^k \rangle_1 - \nabla_k m_1 \langle \rho_1 \rangle_1 \langle \hat{v}_{1i} \hat{v}_1^k \rangle_1 +$$

$$+ \nabla_k (m_1 \langle p_{1i}^k \rangle_1 + m_2 \langle p_{2i}^k \rangle_2) + g_i (m_1 \langle \rho_1 \rangle_1 + m_2 \langle \rho_2 \rangle_2). \tag{3}$$

Here $\hat{v}_{1i} = v_{1i} - \langle v_{1i} \rangle_1$ is the fluctuation of the quantity, i.e., its deviation from the mean value. Furthermore, we neglect the second term in the right side of (3), which is the fluctuating momentum transport.

We write the averaged momentum equation for the liquid phase in the form

$$\partial_t m_1 \langle \rho_1 \rangle_1 \langle v_{1i} \rangle_1 = -\nabla_k m_1 \langle \rho_1 \rangle_1 \langle v_{1i} \rangle_1 \langle v_1^k \rangle_1 + \tag{4}$$

$$+ R_{21i} + \nabla_k \sigma_{1i}^k + m_1 \langle \rho_1 \rangle_1 g_i,$$

$$\sigma_{1ik} = m_1 \langle p_{1ik} \rangle_1,\ R_{21i} = \sigma_{12} \langle p_{1ik} n_1^k \rangle_{12}.$$

Here R_{21i} is a vector governing the force effect of the phase 2 on the phase 1 per unit volume of mixture; the tensor σ_{1ik} determines the stress acting on phase 1 on the surface of a certain mixture volume.

We find the momentum equation for the solid phase by subtracting the momentum equation for the liquid phase (4) from the momentum equation for the mixture (3) and after some manipulation we can write

$$\partial_t m_2 \langle \rho_2 \rangle_2 \langle v_{2i} \rangle_2 = \nabla_k \sigma_{ei}^k + \nabla_k m_2 \langle p_{1i}^k \rangle_1 - R_{21i} + m_2 \langle \rho_2 \rangle_2 g_i, \tag{5}$$

$$\sigma_{eik} = m_2 (\langle p_{2ik} \rangle_2 - \langle p_{1ik} \rangle_1).$$

Here σ_{eik} is the effective stress tensor describing the force interaction between the matrix grains because of their contiguity.

Taking the average of the microdeformation tensor ε_{2ik} with respect to phase 2, we have

$$E_{2ik} = \varepsilon_{eik} + \langle \varepsilon_{2ik} \rangle_2, \tag{6}$$

where E_{2ik} and ε_{eik} are the macrodeformation tensors and effective deformations, respectively [3]. The tensor ε_{eik} is related to the grain displacement with respect to each other.

According to the Maxwell equations, for a nonelectrically conductive medium (2)

$$R_{21i} + \nabla_k \sigma_{1i}^k = - m_1 - \nabla_i \langle p_{01} \rangle_1 - m_1 \nabla_i \langle p_1^m \rangle_1 - \sigma_{12} \langle \hat{p}_{01} n_{1i} \rangle_{12} - \sigma_{12} \langle \hat{p}_1^m n_{1i} \rangle_{12} + m_1 \langle M_1^j \rangle_1 \nabla_j \langle H_{1i} \rangle_1 +$$

$$+ \sigma_{12} \langle M_{1j} \rangle_1 \langle \hat{H}_1^j n_{1i} \rangle_{12} + m_1 \langle \hat{M}_1^j \nabla_j \hat{H}_{1i} \rangle_1 + \sigma_{12} \langle \tau_{1ij} n_1^j \rangle_{12} + \nabla_j m_1 \langle \tau_{1ij} \rangle_1. \tag{7}$$

Here p_1^m is the magnetic addition to the pressure, described by the third term in the right side of the seventh formula (2).

Furthermore, in application to (7) we make the following assumptions: 1) we neglect the change in viscosity η_1 in the averaging volume element; 2) we consider the magnetic permittivity of the fluid not to differ very radically from 1 (as holds in sufficiently strong fields), 3) we assume that the relationships

$$\sigma_{12} \langle \hat{p}_{01} n_{1i} \rangle_{12} = -\chi_{mij} \rho_1 m_1 m_2 \left[\frac{d_1 \langle v_1^j \rangle_1}{dt} - \frac{\partial \langle v_2^j \rangle_2}{\partial t} \right], \tag{8}$$

$$\sigma_{12} \langle \tau_{1ij} n_1^j \rangle_{12} = - A_{ik} \eta_1 m_1 m_2 (\langle v_1^k \rangle_1 - \langle v_2^k \rangle_2)$$

hold for the general case of a statistically inhomogeneous and anisotropic medium.

In connection with assumption 2), we neglect the term in (7) that contains the product of the magnetization fluctuations by the field.

The integral over the interphasal surface $\sigma_{12} \langle \hat{H}_1^j n_{1i} \rangle_{12}$ in (7) is different from zero only when the vector fields H_{1i} and, therefore, also $\langle H_{1i} \rangle_1$ are inhomogeneous in space because the fluctuation evidently equals zero for a homogeneous field. It hence follows that the integral mentioned above is a function of the argument $\nabla_k \langle H_{1l} \rangle_1$ that vanishes together with the argument. Expanding this integral in a series and retaining first order terms in the expansion (under the assumption that $\langle H_{1l} \rangle_1$ varies slowly in space), we have

$$\sigma_{12} \langle \hat{H}_{1j} n_{1i} \rangle_{12} = m_1 m_2 a_{jikl} \nabla^k \langle H_1^l \rangle_1 + \ldots \tag{9}$$

Here the tensor a_{jikl} characterizes the properties of the medium. Analogous expansions can be written down also for the remaining integrals of the fluctuations over the interphasal surfaces. The quantity $m_1 \langle \tau_{1ij} \rangle_1$ is written in the form

$$m_1 \langle \tau_{1ij} \rangle_1 = \eta_1 [\nabla_i m_1 \langle v_{1j} \rangle_1 + \nabla_j m_1 \langle v_{1i} \rangle_1] + \eta_1 \sigma_{12} \langle n_{1i} v_{1j} + n_{1j} v_{1i} \rangle_{12}.$$

We assume that the effective stress tensor is related to the effective strain tensor by the generalized Hooke's law

$$\sigma_{eij} = m_2 \Lambda_{ijkl} \varepsilon_e^{kl}. \tag{10}$$

Here the effective stress tensor should be expressed in terms of the macrodeformation tensor (6) that describes the matrix deformation being observed.

The averaged continuity equation for the phase α has the form

$$\partial_t m_\alpha \langle \rho_\alpha \rangle_\alpha + \nabla_k m_\alpha \langle \rho_\alpha \rangle_\alpha \langle v_\alpha^k \rangle_\alpha = 0 \tag{11}$$

in the absence of phase transitions.

The equations of state for both phases are

$$\rho_\alpha = \rho_\alpha(p_{0\alpha}, T_\alpha).$$

Here $p_{0\alpha}$ is the pressure in phase α without a magnetic field. We define the pressure in a solid body as follows

$$p_{02} = -(1/3)\, p_{02k}^{k} = K_2[\varepsilon_{2k}^{k} - \beta_{2T}(T_2 - T_0)].$$

For small changes in the phase densities, the equations of state can be linearized:

$$\frac{\langle \rho_\alpha \rangle_\alpha}{\langle \rho_{\alpha 0} \rangle_\alpha} = 1 + \beta_{\alpha p} \langle p_{\alpha 0} \rangle_\alpha - \beta_{\alpha T}(\langle T_\alpha \rangle_\alpha - T_{\alpha 0}). \tag{12}$$

We assume that the pressure in each phase equals zero for $T_\alpha = T_{\alpha 0}$.

The equation of compatibility of the phase deformations that describes the change in the porosity m_1 can easily be found by differentiation of both sides of (12) with respect to the time. It follows from this equation that besides the thermal effects the Maxwell stresses in the phases influence the change in porosity.

Taking the average of the heat conduction equation for the phase α and neglecting products of the fluctuations, we have

$$\rho_\alpha c_\alpha [\partial_t m_\alpha \langle T_\alpha \rangle_\alpha - \langle T_\alpha \rangle_\alpha \nabla_k m_\alpha \langle v_\alpha^k \rangle_\alpha - \langle T_\alpha \rangle_\alpha \partial_t m_\alpha + \nabla_k m_\alpha \langle v_\alpha^k \rangle_\alpha \langle T_\alpha \rangle_\alpha + \tag{13}$$

$$+ \sigma_{12} \langle n_\alpha^k (v_{\alpha k} - u_k) T_\alpha \rangle_{12}] = \nabla_k (m_\alpha \varkappa_\alpha^{kl} \nabla_l \langle T_\alpha \rangle_\alpha) + \sigma_{12} \langle n_\alpha^k \varkappa_\alpha \nabla_k T_\alpha \rangle_{12}.$$

In the absence of phase transitions we set $v_{\alpha k} = u_k$.

In connection with the fact that the fluid velocity on the pore surface equals the velocity of this surface, the heat transfer of the pore surface with fluid occurs by ordinary heat conduction. It hence follows that the last term in the right side of (13) is the interphasal heat transfer $Q_{\alpha\Sigma}$ [3]:

$$Q_{\alpha\Sigma} = \sigma_{12} a_1^{-1} \varkappa_\alpha \mathrm{Nu}_\alpha (\langle T_\Sigma \rangle_{12} - \langle T_\alpha \rangle_\alpha).$$

According to the last boundary condition for temperature (2), the equality $Q_{1\Sigma} = -Q_{2\Sigma}$ holds.

Let us turn to the derivation of the averaged Maxwell equations. We determine the magnetic field intensity averaged over the mixture $\langle H_k \rangle$ by the relationship

$$\langle H_k \rangle = \sum_{\alpha=1}^{2} m_\alpha \langle H_{\alpha k} \rangle_\alpha.$$

The induction $\langle B_k \rangle$ averaged over the mixture is determined analogously. In the general case of a statistically anisotropic medium, the intensity and induction are associated by the relationship $\langle B_i \rangle = \mu_{cij} \langle H^j \rangle$. Here the magnetic permittivity tensor of the mixture μ_{cij} is assumed a known function of the porosity, density and temperature of the phases, as well as of the averaged field $\langle H_k \rangle$.

Averaging the Maxwell equations (2) with respect to the phases and combining, and taking account of the boundary conditions, we find an equation for the field $\langle H_i \rangle$:

$$\nabla_i \mu_c^{ij} \langle H_j \rangle = 0, \quad \varepsilon^{ijk} \nabla_j \langle H_k \rangle = 0. \tag{14}$$

Equations must also be derived to find the magnetic field averaged over the pore volume $\langle H_{1i} \rangle_1$, in the momentum equation for the liquid phase (4). Averaging the Maxwell equations (2) with respect to the phase 1 and transforming the integrals over the interphasal surface by using the local coordinate system in the averaging space, with the assumptions enumerated earlier taken into account, we find equations for the field $\langle H_{1i} \rangle_1$:

$$m_1 [\nabla_k \langle H_1^k \rangle_1 + 4\pi \nabla_k \langle M_1^k \rangle_1] + \sigma_{12} \langle n_1^k [\hat{H}_{1k} + 4\pi \hat{M}_{1k}] \rangle_{12} = 0, \tag{15}$$

$$m_1 \varepsilon^{ijk} \nabla_j \langle H_{1k} \rangle_1 + \sigma_{12} \varepsilon^{ijk} \langle n_{1j} \hat{H}_{1k} \rangle_{12} = 0.$$

The integrals over the interphasal surface in (15) are functions of the arguments $\nabla_i \langle H_{1j} \rangle_1 + 4\pi \nabla_i \langle M_{1j} \rangle_1$ and $\nabla_j \langle H_{1k} \rangle_1$, respectively, and can be approximated by the first terms in their series expansions, analogously to the relationship (9).

Let us note that the porosity m_1, which varies during filtration, is in (15).

The magnetic field averaged over phase 2, which is in the solid-phase momentum equation (5), is found from the relationship

$$\langle H_2^k \rangle_2 = m_2^{-1} [\langle H^k \rangle - m_1 \langle H_1^k \rangle_1].$$

It should be noted that since the porosity, magnetic field intensity, and other quantities are in all the equations describing the filtration, the whole system of equations must be solved to find these quantities. For instance, just some of the Maxwell equations are now inadequate for finding the magnetic field.

The boundary conditions for $\langle H_i \rangle$ and $\langle B_i \rangle$ reduce to the continuity of the tangential component of the field $\langle H_i \rangle$ and the normal component of $\langle B_i \rangle$ in the case of a nonelectrically conductive medium.

In conclusion, we note that evaluation of the elasticity coefficients for the matrix frame consisting of periodically arranged balls of identical radius is possible, in principle, by using the results of solving the Hertz problem for two contiguous spheres [5, p. 45].

NOTATION

∇_i, covariant derivative; $\partial_t = \partial/\partial t$; $\langle \ldots \rangle$, average over the mixture; $\langle \cdots \rangle_\alpha$, average over the phase α; $\langle \cdots \rangle_{12}$, average over the interphasal surface; u_i, velocity of interphasal surface motion; σ_{12}, interphasal surface per unit volume; $n_{\alpha i}$, normal to the interphasal surface, external for the phase α; $v_{\alpha i}$, velocity of the medium; ρ_α, density; g_i, free-fall acceleration; $p_{\alpha ik}$, stress tensor; $H_{\alpha k}$, $B_{\alpha k}$, magnetic field intensity and induction; p_1, total pressure; p_{01}, pressure without the field; η_1, viscosity; c_α, specific heat; T_α, temperature; g_{ik}, metric tensor; λ_2, μ_2, Lamé coefficients of the matrix material; h_k, displacement vector in the matrix; β_T, coefficient of thermal expansion; $\varkappa_\alpha$, heat-conduction coefficient; $\{A\}$, discontinuity in the quantity A on the interphasal surface; χ_{mij}, tensor of apparent mass coefficients; A_{ik}, friction coefficient tensor; m_1, matrix porosity; $m_2 = 1 - m_1$; $m_2\Lambda_{ijkl}$, elastic modulus tensor of the porous matrix frame; K_2, multilateral compression modulus; $\varkappa_\alpha^{kl}$, heat conductivity tensor of the phase α; ε_{ijk}, Levi–Civita tensor; Nu_α, Nusselt number; T_Σ, interphasal boundary temperature; a_1, average characteristic pore dimension.

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